## **REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS**

## [C, D, E, L, X].—C. W. CLENSHAW, Chebyshev Series for Mathematical Functions, Mathematical Tables, v. 5, National Physics Laboratory, Her Majesty's Stationery Office, London, 1962, iv + 36 p., 27.5 cm. Price 12s.6d.

In an introductory section the author states the two-fold purpose of this volume, namely, to present tables, mostly to 20 decimal places, of the coefficients in the Chebyshev expansions of a number of the more common mathematical functions, and to set forth the basic techniques for evaluating and manipulating such series.

The remaining text consists of sections devoted to: the fundamental properties of Chevyshev polynomials (with particular reference to a discussion by Lanczos [1]); the alternative methods used in the calculation of Chebyshev coefficients (use of the orthogonal properties of integration and of summation, rearrangement of series, and solution of differential equations); a description of the tables and their preparation, mainly by solving the appropriate differential equations; the application of the tables and the use of Chebyshev series; and a critical comparison of Chebyshev series with alternative forms of storing a table-equivalent in a computer (these forms include explicit polynomials, "best" polynomials, and rational functions).

A bibliography of 28 books and papers is included, followed by two appendixes: one on the spelling of "Chebyshev," and the other on the normalization of Chebyshev polynomials.

The body of this work consists of seventeen tables of Chebyshev coefficients, given to 20 decimal places except for the last table, which gives 14 places. The functions considered include the trigonometric functions sine, cosine, and tangent; the inverse functions  $\sin^{-1}x$ ,  $\tan^{-1}x$ ; the exponential function; the logarithmic function  $\ln(1 + x)$ ; the inverse hyperbolic sine; the Gamma function: the error function; the exponential integral; and both regular and modified Bessel functions of orders 0 and 1, together with auxiliary functions.

The numerous summation checks of the Chebyshev coefficients that are included in the tables inspire confidence in the accuracy of these extensive results. This reviewer has, moreover, found that Clenshaw's values of  $J_{2k}(n\pi/2)$  agree with similar data of Owen R. Mock deposited in the UMT file (see Review 118, MTAC, v. 9, 1955, p. 223).

This excellent set of tables appears to be the most extensive and elaborate compilation of such coefficients that has yet been published.

J. W. W.

1. NATIONAL BUREAU OF STANDARDS, Tables of Chebyshev Polynomials, Applied Mathematics Series No. 9, U. S. Government Printing Office, Washington, D. C., 1952.

2 [F].—J. C. P. MILLER, *Table of Least Primitive Roots*, 3000-page manuscript in possession of the author at The University Mathematical Laboratory, Cambridge, England; one copy thereof deposited with the Royal Society of London;

a second copy temporarily in the custody of Professor D. H. Lehmer, University of California, Berkeley 4, California, prior to deposit in UMT File.

This extensive manuscript table lists for each of the 150,000 primes p from 3 to 2,015,179, inclusive, the following four integers: g, the least positive primitive root; h, the least positive prime primitive root; g', where -g' is the negative primitive root of least modulus; and h', where -h' is the negative prime primitive root of least modulus.

When p is of the form 4k + 1, g' and h' are equal to g and h, respectively, and are not explicitly tabulated; furthermore, h is not tabulated for such primes unless it differs from g. On the other hand, for primes of the form 4k - 1, all four data are shown explicitly.

The primes p are listed 50 to a page, the last three figures of each appearing in the argument column; to these figures there must be added the multiple of a thousand listed at the top of the column, with an increment of a thousand if the thousands digit has changed since the top of the column.

The author has informed this reviewer that this table was computed in about 50 hours on EDSAC 2 by means of a program prepared by M. J. Ecclestone. As the result of built-in program checks and visual inspection of all the printed output, this table is considered to be very reliable.

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3 [F. G, I, X].—JOHN TODD, Editor, A Survey of Numerical Analysis, McGraw-Hill Book Co., 1962, 23.5 cm., xvi + 589 p. Price. \$12.50

In 1957 the National Bureau of Standards, with support from the National Science Foundation, offered a course designed for mature mathematicians and aimed at arousing their interest in the practice and theory of computing. This book is made up of lectures prepared for this course, except for two of the chapters which were prepared in collaboration with lecturers who spoke at a similar course given in 1959.

Chapters include one on "motivation," and topics range from simple interpolation, through matrices and differential equations, to the application of functional analysis to numerical analysis, with added chapters on discrete problems, number theory, and linear estimations. Contributors are John Todd, Olga Taussky, Morris Newman, Harvey Cohn, Philip Davis, Marvin Marcus, Henry A. Antosiewicz, Walter Gautschi, David Young, Hans Bückner, Werner Rheinboldt, Marshall Hall, and Marvin Zelen.

Although there are substantial differences among chapters as to depth and sophistication, and the coverage is not uniform, nevertheless there is much more coherence and coordination than one might expect, and the coverage is quite extensive. Rather long lists of references are included in each chapter, although usually the authors made little effort to bring them up to date (i.e., later than 1957). It is regrettable that publication was so long delayed, but the book is a valuable contribution, nevertheless.

A. S. HOUSEHOLDER

EDITORIAL NOTE: A list of corrections to this volume has been compiled by the publisher.